

The Impact of Downside Beta on Stock Price Behavior in Pakistan

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ABSTRACT

Purpose- The purpose of this study is to construct a portfolio based on downside beta as a measure of downside risk. The analysis uses data from all listed companies on the Pakistan Stock Exchange (PSE) over the period from January 2000 to December 2021.

Study Design/Methodology/Approach - The study employs a rolling window approach with 36 consecutive months to estimate downside beta values for portfolios ranked from lowest to highest downside beta. Portfolio returns are evaluated using the Generalized Method of Moments (GMM) in conjunction with the Capital Asset Pricing Model (CAPM) and the Fama-French three- and five-factor models. A six-month Treasury bill rate is used as the risk-free rate.

Findings- The study reveals a significant spread between the downside beta values of the constructed portfolios, confirming that downside risk can be effectively captured through this approach. The hypothesis of equal mean downside beta across the portfolios is rejected, indicating that portfolios based on downside beta exhibit different risk-adjusted returns. Furthermore, the rejection of the alpha equal to zero hypothesis suggests that these risk-adjusted returns are statistically significant. However, the study finds that lower partial moments do not fully explain the variation in downside risk.

Practical Implications- These findings are particularly relevant for investors focused on managing downside risk. The study offers alternative methods for constructing portfolios that account for downside risk, which could enhance risk management strategies. However, the reliance on historical data from the PSE may limit the generalizability of these findings to other markets.

Originality/Novelty - This study contributes to the literature by constructing portfolios grounded in downside beta as a measure of downside risk, offering a novel approach to portfolio formation and risk-adjusted return evaluation. It highlights the importance of incorporating downside risk in portfolio management and presents an innovative application of downside beta within the context of emerging markets.

Keywords: CAPM, Portfolio, Downside Risk, Efficient Market Hypothesis, Semi-variance

1 | INTRODUCTION

For many decades, many developments have been occurring in the field of asset pricing. The main aim of this development is to improve the decision-making process of an investor. Through efficient decision

making the profitability of an investor even in the worst market condition can be maximized. Further risk and returns estimation for the portfolio assured appropriate returns in the market. These theories of risk and return have performed their role in developing an understanding of stocks, bonds, and the prices of stocks. It is also subjective to influence the field of corporate finance and macroeconomic indicators. The capital assets pricing model of [Sharpe \(1964\)](#) and [Lintner \(1965\)](#) celebrated almost 44 birthdays. But yet the CAPM is a widely used model to predict the return risk relationship. If we go back to history, we come to know that there were so many attempts made to improve the work of CAPM. As we discussed CAPM is a widely used model to assess the security price concerning their risk but at the same time, it ignores the safety rule of investors. Therefore, the downside capital asset pricing model is being introduced to check its reliability in the context of the Pakistani market to provide the price of stock when the market goes down as compared to its normal behavior.

CAPM is completely based on [Markowitz \(1952\)](#), which predicts the price based on risk to a market portfolio it theoretically assumes that more risky assets require more risk premium for the investor it can be represented by the following equation.

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

While this is a basic model of CAPM there is so much criticism being made to this model as it cannot provide safety to investors in bad market time. Therefore D-CAPM is used for the estimation of risk and then to compare with the risk measure of normal CAPM.

1.1 | Problem Statement

The Capital Asset Pricing Model (CAPM) is an introductory model for assessing expected returns based on market risk. However, CAPM's dependence on the mean-variance method, which adopts normal and symmetrical return distributions, has been criticized for its incapability to capture downside risk, mainly in volatile and irregular markets like Pakistan ([Al-Yahyaee et al., 2019](#)). Research has revealed that during market downturns, CAPM may underestimate risk, possibly leading to suboptimal investment choices for risk-averse investors ([Santi, 2023](#)). Accordingly, there is an amplified interest in alternative models, such as the downside CAPM (D-CAPM), which uses downside beta¹ as a degree of risk to capture investors' aversion to losses more effectively.

Studies in current years have established that downside risk aspects may offer a more reliable estimate of expected returns in emerging markets, where return distributions often diverge from normality ([Chung & Yoon, 2020](#); [Gkillas et al., 2021](#)). The D-CAPM has gained attention as a model that considers semi-variance rather than total variance, positioning better with the preferences of risk-sensitive investors. For example, the study by [Chung and Yoon \(2020\)](#), outlined the significance of downside beta in capturing the unique risks present in emerging markets during periods of economic downturn. This study aims to examine the effectiveness of D-CAPM within the Pakistani stock market context, assessing whether downside beta provides more accurate risk-adjusted returns compared to traditional CAPM.

1.2 | Historical Development

¹**Downside beta** is a measure of an asset's sensitivity to negative market movements, contrasting with outdated beta, which measures responsiveness to overall market fluctuations. It precisely captures how much an asset's returns decay in response to downturns in the market, providing insight into the risk profile of investments during adverse conditions

1.2.1. CAPM Development

The Capital asset pricing model (CAPM) has faced some serious empirical difficulties for the last many decades. More specifically CAPM has failed to explain the return of many equities based single factor of risk mentioned by ([Basu, 1977](#)). The same study was done by [Banz \(1981\)](#), on the different sample sizes and produced the same Empirical results. Moreover, [Jegadeesh and Titman \(1993\)](#) argued that CAPM only provides a single factor for risk therefore, it ignores the measures of risk while predicting the returns. The most recent attempt for analyzing risk and return, principally in diversified portfolios is done by (CFA Institute, 2023). Current research highlights its adaptability, such as through variants like the intertemporal CAPM and downside CAPM, which address CAPM's limitations in accounting for time-varying risks and asymmetric risk preferences (MDPI, 2023). While many other authors describe the failure of CAPM as they said that it provides biased empirical methodology as it provides no base for risk explanation. Eventually, ([Fama & French, 1993](#)); ([Fama & French, 2015](#)) added more risk factors to capture more efficient results.

For the last thirty years, many researchers and academics have been in the process of balancing out the merits and demerits of CAPM while considering whether beta provides a reasonable measure of risk or not. This research ends with discussing with the help of some empirical evidence to compare the capability of beta to describe the asset return with their alternative risk variables. However, these debates missed where an investor uses beta as a risk measure which depends upon the Mean-variance behavior in other simple words investor uses equilibrium to maximize the utility to depend upon the mean-variance approach of return of their portfolio. However, the variance of return is questionable at least for two reasons. First, it is a suitable risk measure only when the underlying distribution is symmetric. Second, it can be used as a measure of risk if only distribution is normally distributed of stock returns. Though both symmetry and normality are questionable are subjected to much research.

1.2.2. Evolving of Downside Beta

Semi semi-variance approach of return on the other hand is the most suitable option while return is not normally distributed and not symmetric as well. Moreover, it is also suitable for many other reasons to measure risk. The first investor always doesn't care for upside beta because during this market growing at a good pace it normally does earn something for their portfolio. On the other hand, downside variation does bother investors and they only dislike downside beta and want its safety first. The second semi-variance approach altogether combines the variance and skewness information in one piece of information and enables us to use the one-factor model to calculate the required rate of return and adjust risk accordingly ([Bokovnya et al., 2020](#)).

Moreover, the semi-variance approach of return can be applied to create an alternative hypothesis based on mean-semi variance behavior (MSB) as discussed by [Estrada \(2000\)](#), who proves that MSP is perfectly correlated with expected compound returns. Consequently, it can be defended on the same line as [Levy and Markowitz \(1979\)](#) defended mean-variance behavior.

In this Independent study, I am going to propose another alternative measure of risk for diversified investors and this downside beta and pricing of asset model. I also include evidence of the Pakistan stock market to check the downside beta model and whether it provides sufficient risk information.

The purpose of this independent study is to check and analyze the empirical performance of CAPM concerning DCAPM meaning the downside capital asset pricing model. As we know CAPM is the most widely

used model in the field of empirical asset pricing but after having an eye on some research we come to know that CAPM is unable to provide the required rate of return to the investor while the distribution is not normal. Therefore, it becomes quite difficult for an investor to make an effective portfolio selection to maximize his utility. Therefore, the paper aims to provide some pieces of evidence that the downside beta model provides a safety measure to investors during the market's worst time so that they can marginalize their return according to market risk. In most cases downside risk is one of the most first preferences of an investor to predict the required return while CAPM provides information based on the Mean-Variance approach meanwhile DCAPM focuses on a variance approach which helps to measure relative risk concerning falls.

1.3 | Mean-Variance Approach

It is very important to discuss the mean-variance approach concerning empirical asset pricing. In the mean-variance² approach, the whole satisfaction of an investor (U) depends upon the Variance σ_i and Mean μ_i therefore $U_i = U(\sigma_i, \mu_i)$. In this structure, the risk of individual asset return is measured by the standard deviation of return (σ_i) which is mentioned below.

$$\sigma_i = \sqrt{E[(R_i - \mu_i)^2]},$$

In this formula, R_i and μ_i are the return and mean of asset i while an asset is an individual asset then the risk of the portfolio is measured by the covariance with the market portfolio (σ_{iM}) which is mentioned below.

$$\sigma_{iM} = E[(R_i - \mu_i)(R_M - \mu_M)]$$

In this equation, M refers to the index of the market portfolio. Here covariance is both scales dependent and unbounded therefore their interpretation is not simple and straight. So, it is more appropriate to divide it by the product of the standard deviation of return σ_i and standard deviation of market portfolio return, thus getting the correlation of asset i to market portfolio (ρ_{iM}) which is given below:

$$\rho_{iM} = \frac{\sigma_{iM}}{\sigma_i \cdot \sigma_M} = \frac{E[(R_i - \mu_i)(R_M - \mu_M)]}{\sqrt{E[(R_i - \mu_i)^2]} \cdot \sqrt{E[(R_M - \mu_M)^2]}}$$

To get β the covariance of asset i can be divided by the variance of the market portfolio by doing that we can have beta as follows.

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{E[(R_i - \mu_i)(R_M - \mu_M)]}{E[(R_M - \mu_M)^2]}.$$

² The mean-variance approach, developed by Harry Markowitz in the 1950s, is a foundational theory in modern portfolio management. It provides a framework for constructing an investment portfolio that aims to maximize returns for a given level of risk or, conversely, to minimize risk for a desired level of expected return.

Eventually, this beta of asset i mean this beta provides the investor with a risk associated with the return of markets. This beta can also be explained as $\beta_i = (\sigma_i/\sigma_m) \rho_{iM}$ this is most frequently used as a measure of risk to market on equity so this beta is regarded as CAPM model beta which can be expressed as follows:

$$E(R_i) = R_f + MRP \cdot \beta_i,$$

This is the general CAPM model which is based on a pure mean and variance approach where R_f is the risk-free rate while MRP is the market risk premium which can be obtained by $(R_m - R_f)$ where R_m is the market required return and β_i is measure of risk to market. Moreover, it is argued that this model is only useful when the return of asset and market are normally distributed and symmetric while in the not normal distribution its results are not efficient therefore, we are approaching the semi-covariance or downside beta approach. Similarly, it is also unbounded and scale-dependent thus it should be standardized by dividing it with a product of semi-deviation of asset i return and semi-deviation of market portfolio return to get the correlation between them.

$$\Theta_{iM} = \frac{\Sigma_{iM}}{\Sigma_i \cdot \Sigma_M} = \frac{E\{\min[(R_i - \mu_i), 0] \cdot \min[(R_M - \mu_M), 0]\}}{\sqrt{E\{\min[(R_i - \mu_i), 0]^2\} \cdot E\{\min[(R_M - \mu_M), 0]^2\}}}.$$

Eventually, the co semi variance of further be divided by the semi-variance of the market to achieve the downside beta of asset i return which can be expressed as:

$$\beta_i^D = \frac{\Sigma_{iM}}{\Sigma_M^2} = \frac{E\{\min[(R_i - \mu_i), 0] \cdot \min[(R_M - \mu_M), 0]\}}{E\{\min[(R_M - \mu_M), 0]^2\}}.$$

By this formula, we can calculate the downside beta for the asset i return and their model can be written as:

$$R_i = R_f + MRP \cdot \beta_i^D$$

By contrasting the model of the mean-variance approach beta has been replaced by the downside beta in this model.

2 | LITERATURE REVIEW

2.1 | The Risk of Security

[Sortino and Price \(1994\)](#) discussed a brief understanding of the concept of risk. It is argued that risk and uncertainty are not the synonym for each other. Rather risk and returns are not detachable in the scenario of uncertain return conditions. Indeed, rewards are incorporated with the specific risk for the investors ([Bokovnya et al., 2020](#)). For an investor, there is one goal to achieve to attend the Minimum acceptable returns (MAR). If the returns fall below the MAR, then there are uncertain conditions and bad outcomes. The second condition includes when this return lies above MAR then for investors it is a good outcome ([Guo et al., 2020](#)). Therefore, returns that are below MAR are linked to risk. Therefore, the risk is only connected to returns which shortfall

the MAR. Moreover, there is some probability associated with outcomes of risk that's called probability distribution. There is also uncertainty connected to the outcomes as well.

The minimum acceptable return (MAR) line differentiates between advantageous and contrary volatility, with deviations above MAR considered rewards and those below as downside risks ([Ang et al., 2006](#); [Sortino & Satchell, 2001](#)). Returns above this line represent gains that exceed the minimum acceptable threshold, while volatility below it indicates risk, emphasizing the need to analyze return shapes and patterns [Guo et al. \(2020\)](#). Although the precise nature of uncertainty is inherently difficult to determine, estimating these patterns helps investors assess potential risks and rewards more effectively

2.2 | Reference Point for Downside Risk

Figure 1

Returns for Different Managers

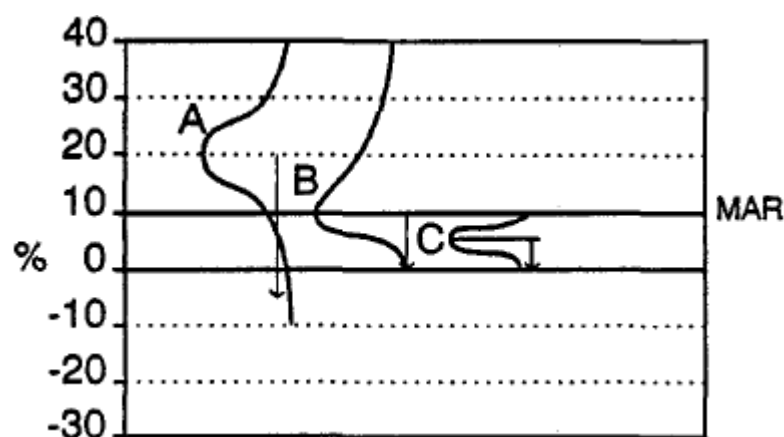


Figure 2 shows the different patterns of returns for three managers A, B, and C. These all are the possible shapes of different returns at different periods. Few have risks while others do not. Because standard deviation is used as the dispersion of returns with both sides of the mean. It cannot differentiate between good dispersion and bad dispersion. Standard deviation provides equal risk to all assets. Consequently, researchers argued that for a better measure of risk that's called downside dispersion. In Figure 2 if we want to calculate the risk then we first have to assign the reference point for each asset to measure downside risk. To avoid any sort of confusion we call it downside deviation (DD). To analyze the difference between the standard deviation and downside deviation let's consider Figure 1. Manager A constructed the portfolio and invested in a well-diversified portfolio & Manager B also inverted a similar sort of portfolio but by removing some lower side risk short. While Manger C invested in the short-term treasury bills. According to the standard deviation approach, Manager C took the least risk. Here Risk refers to not earning the mean of the Treasury bill but the question raises what should be at stack? And it does not capture the stack. What is more concerned is attaining the minimal accepted returns of 10 % so this approach to Manager B is less risky is as Manager C as their investment is at least touching the MAR line.

There is a criticism made to the argument that if the distribution of risk is asymmetric then downside risk is the same standard deviation but they argued that most of the distribution is not asymmetric and if it's then investors should not measure the risk with their mean for every asset. Let's consider Figure 2 here there are

three managers each with a different reference point to measure the risk. Manager A uses their mean as their benchmark to measure the risk, Manager B uses MAR 10 % as a reference point to measure the deviation. Manager C uses risk-free rate to calculate the risk. The downside arrows show the distance of the measure of risk for each asset. If we are using the approach of Standard deviation then Manager B incorrectly reported as riskier than manager C as their distance from the MAR line to the downside risk more as compared to Manager C, While Manager C does not have a chance to touch the line of MAR therefore, he has nothing to lose as T bill are riskless assets other than defaults.

[Sortino and Price \(1994\)](#) stated that the reference point of each asset is considered to be a mandatory component to measure the downside deviation. He argued that the reference point is the stated aim or goal of an investor in terms of return and risk which he wanted to be attained. This means that maximization of profit with subject to falling short behind the Minimal accepted return. If the MAR 10 % is the reference point for the manager B & C then C is considered to be more risk than C as its whole distribution is shortfall than the MAR 10 %.

2.3 | Appropriate Model for Risk & Return.

Literature is full of debates and explanations concerning risk and return measure models and different scholars have already been defining arguments to provide the best measure of risk. Therefore, we are starting with [Markowitz \(1952\)](#), who started the work of portfolio management. In 1952 Markowitz defined the optimal portfolio concept. [Markowitz \(1959\)](#) conclusion is that the best portfolio is that which provides the optimal combination of risk and return for the investor according to his tolerance capacity. Eventually, Markowitz developed the mean-variance framework to establish an efficient portfolio. He provides in his mean-variance approach a preference for higher return over lower return and secondly stability over volatility. These are the two main objects that provide and form efficient portfolios for model investors ([Schoenmaker & Schramade, 2023](#)).

Consequently, [Tobin \(1958\)](#) formulated liquidity preference theory in addition to [Markowitz \(1952\)](#) mean-variance. Tom connected the preference of investors to the indifference curve. [Thomas \(1996\)](#) concluded that investors presumed his preference between expected risk and return which is represented by the indifference curve. These theories of Markowitz and Tom have some basic assumptions. [Maneemaroj et al. \(2021\)](#) discover the relationship between expected returns and related risks, providing an understanding of suitable expectations for investors.

2.4 | Formulation for the Measurement of Downside Risk (Beta)

[Hogan and Warren \(1974\)](#) followed the work of [Markowitz \(1959\)](#) on semi-variance and explained the target average return concept. They said that if the return is above the target average return, it constitutes no risk for the investors. The early work of [Markowitz \(1959\)](#) and [Roy \(1952\)](#) advised different methods of semi-variance risk matrices but they also added that these methods are quite time-consuming because they require a very high order of calculation. [Markowitz \(1959\)](#) expressed that the popularity of the mean-variance approach is that it is easily calculable and managed and their interpretation can be understood by the common public very easily. More specifically it is familiar and cost effective as well. However, he proclaimed that the semi-variance approach provides more accurate results than the mean-variance approach but their calculation is a bit difficult. The main reason for its difficulty is that it involves algorithms and other calculations. It was

considered very difficult in the past when computational techniques were not available but nowadays it is possible to calculate such vast and huge calculations and reach some effective results ([Liu, 2023](#)).

Moreover, the most important study has been done by Black, Jensen, schools (BJS), and Fama and Macbeth which is still very well applicable at present to financial econometrics. [Black \(1972\)](#) tested CAPM for the New York Stock Exchange from the period of 1926-1966 testing whether the intercepts are zero on market beta on cross-section regression of excess return. He found there is a positive relationship between risk and return. [Blume and Friend \(1973\)](#) and [Fama and MacBeth \(1973\)](#) found similar results. They added that change in covariance has an impact on the results of risk and return.

CAPM addressed the distribution if it is normal but the distribution of return is skewness and kurtosis therefore it is argued by some scholars that CAPM is unable to capture the effective or non-normal distribution. [MacKinlay \(1997\)](#) explained that the problem of not normal data can be solved by the general method of moment³ (GMM). By using this method on non-normal data efficient results can be obtained from ([Ayub et al., 2020](#)).

Given the safety rule first [Bawa and Lindenberg \(1977\)](#) extended the CAPM model to DCAPM to provide efficient results while the market is underperforming and it is written as follows:

$$\beta_{im}^{BL} = \frac{E[(R_i - R_f) \min(R_m - R_f, 0)]}{E[(\min(R_m - R_f, 0))^2]}$$

In the numerator, there is co a semi-variance of asset I and a semi-variance of market return only those which are in negative sign. While it is also argued that that risk downside also fell below target return [Harlow and Rao \(1989\)](#) suggest that a risk-free rate can be replaced by average market return and written as follows:

$$\beta_{im}^{HR} = \frac{E[(R_i - \mu_i) \min(R_m - \mu_m, 0)]}{E[(\min(R_m - \mu_m, 0))^2]}$$

[Harlow and Rao \(1989\)](#) determined a framework to measure the downside risk. They made a change of Bawa and Lindbergh downside beta as they replaced the risk-free rate with mean μ or of respective security and market. They argued that in light of their empirical finding, it means an empirically more appropriate target rate as compared to a risk-free rate. By establishing the mean lower partial moments (MLPM) framework they use a sample of returns of security to test it. Consequently, they found that the results of the Mean as the target rate for beta is better and could not be rejected therefore, he replaced the risk-free rate with the mean of security for the calculation of downside risk. After [Harlow and Rao \(1989\)](#), [Estrada \(2000\)](#) made some amendments to the formulation of downside beta and produced the following framework.

³he Generalized Method of Moments (GMM) approach is an econometric technique used to estimate parameters of statistical models by utilizing sample moments (such as means and variances) and optimizing the fit of the model to these moments, allowing for efficient estimation even with potentially non-standard distributions

$$\beta_{im}^E = \frac{E[\min(R_i - \mu_i, 0) \min(R_m - \mu_m, 0)]}{E[(\min(R_m - \mu_m, 0))^2]}$$

[Estrada \(2000\)](#) argued that there are three main differences between the [Bawa and Lindenberg \(1977\)](#) formula and his framework of downside risk. The first difference is that Estrada only calculates returns when $R_i < \mu_i$ and $R_m < \mu_m$, While [Bawa and Lindenberg \(1977\)](#) calculate the risk when $R_i < \mu_i$, $R_m < \mu_m$ but it reduces the risk when $R_i > \mu_i$. Second, they used risk-free rate as their benchmark for downside beta while Estrada uses the mean of respective assets. We have applied the Estrada formula for the calculation of downside risk in this research paper. Because through this formula, we can calculate the downside beta when returns are less than their mean value.

The argument between CAPM and DCAPM does not direct that CAPM is a failed model or useless. It means that DCAPM provides more accurate results in the presence of skewness and normality of data. While CAPM is still a useful model that provides investors with an efficient result data is normally asymmetric [Menon et al. \(2023\)](#).

Furthermore, while moving toward the GMM General method of moments it is necessary to understand the review of how it came to exist [Fisher and D'Alessandro \(2021\)](#). The single-factor model of CAPM goes under tremendous scrutiny and debate and it is argued that the single-factor model is not sufficient enough to capture cross-section variation. [Sharpe \(1964\)](#) and [Lintner \(1965\)](#) test in their article that alone beta is not enough to capture the variation of cross-section. A variety of research is provided that size, book-to-market, price, and price-earnings ratio also count to explain the variation of cross-section return and risk.

[Fama and French \(1996\)](#) provide the significance of SMB (the difference between a portfolio of small stock return and a portfolio of large stock return) and HML (the difference between high book-to-market value and a portfolio of low book-to-market value).

Numerous empirical findings argue to reject the single-factor model of CAPM. Firstly, the CAPM single-factor model is rejected when the portfolio uses a proxy and the market is inefficient see, [Roll \(1977\)](#). [Roll and Ross \(1994\)](#) also showed that even a very small deviation of in efficiency provides an insignificant relation between risk and return. In this paper we used skewness which is the third order of moments provide you an understanding of the cross-section of variation. [Kraus and Litzenberger \(1976\)](#) provided the nonlinear model in which they used skewness to explain the variation while he said that positive skewed data may be liked by investors as it has the most return on the positive side. Our work in this paper is slightly different from Kraus and Litzenberger's as we they unconditional skewness while we are going to use conditional skewness to provide the value at risk (VAR) our focus is to explain the cross-section variation of stock return in case of downside beta. Our work follows the work of [Harvey and Siddique \(2000\)](#) they found co-skewness and co-kurtosis on NYSE and formed the portfolio based on low to high Co skewness and Co-kurtosis. In this paper doing the same operation with Pakistan stock exchange listed companies as well as a mean semi-variance approach, [Khurram et al. \(2021\)](#).

3 | METHODOLOGY & DESIGN

3.1 | Data Collection and Procedure

This Research includes the data set of all companies listed in the Pakistan Stock Exchange. The sample consists of data from January 2000 to December 2021 available in Thomson Reuters's data stream. As we have included all the companies listed in the Pakistan Stock Exchange, therefore our data set is free from any bias. Data sets pass through so many scrutiny techniques to get only reliable data. For this instance, we have eliminated the companies that do not have the return for at least 36 consecutive months. We require 36 consecutive months of returns to run a rolling window operation to estimate downside beta values.

The first phase of the procedure includes the calculation of returns through the opening and ending prices of each company. These returns are denoted by the symbol R_i . Karachi Stock Exchange returns are used as market returns denoted by the symbol R_m . Six-month T-Bill rate is used as a risk-free rate (R_f). We have followed the methodology of (Harvey & Siddique, 2000). By using the rolling window approach, we calculated the values of the downside beta. Estrada (2000) discusses several methods to estimate the downside beta and proposes the following formula to estimate the downside risk as well as downside beta.

$$\beta_{im}^E = \frac{E[\min(R_i - \mu_i, 0) \min(R_m - \mu_m, 0)]}{E[(\min(R_m - \mu_m, 0))^2]}$$

Here R_i is the Return of security i while μ_i is the mean return of security i and it's multiplied by the difference of market return and its mean in addition to the restriction of min imposed to get the downside beta. Restriction min refers to calculating the minimum value between ($R_m - \mu$ and 0) meaning that it will calculate only if the value is negative otherwise, it marks the whole value as zero. With the help of this formula, we will gate downside betas rather than normal beta, and based on this beta, we formulate 10 portfolios. These 10 portfolios are formulated based on lower to higher betas values. Arrangement of the portfolio is done from lowest to highest beta values. P1 (portfolio 1) has the lowest downside beta while P10 (the tenth portfolio has the highest downside beta. After this, we apply the beta of this portfolio to the CAPM model and DCAPM model and analyze which portfolio model provides us with better return results. Mentioned below is the CAPM model.

$$R_{i,t} - R_t^f = \alpha_i + \beta_{i,MKT} (R_{m,t} - R_t^f) + \varepsilon_{i,t}$$

Through this model, Johnsen alpha is calculated also called the CAPM alpha. This alpha is calculated for both the returns Value weighted (VW) and equally weighted (EW). Moreover, we have calculated the Fama French alpha through 3 factors as well as 5 factors to compare the alpha. We have used the following equation for the Fama French model for three and five factors. The three-factor French Model given by Fama and French (1993) is as follows.

$$R_{i,t} - R_{ft} = \alpha_{(3\text{-factor})} + \beta_{i,MKT} (R_{m,t} - R_{ft}) + \beta_{i,SMB} SMB + \beta_{i,HML} HML + \varepsilon_{i,t}$$

Where $R_{i,t}$ is the return of portfolio i at time t R_f is the risk free rate which 6-month t bill rate is used in its $R_{m,t} - R_{ft}$ is risk premium. Whereas SMB is the size factor and HML is the value factor. $\alpha_{(3\text{-factor})}$ Is Fama

French 3-factor alpha $\beta_{i,SMB}$ is the risk coefficient of the size factor while $\beta_{i,HML}$ is the coefficient for the value factor. We have also estimated the Fama French five-factor model through the following equation.

$$R_{i,t} - R_{ft} = \alpha_{(5\text{-factor})} + \beta_{i,MKT}(R_{m,t} - R_{ft}) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + \beta_{i,RMW}RMW + \beta_{i,CMA}CMA + \varepsilon_{i,t}$$

In this equation, RMW is the investment factor, while CMA is the profitability factor. $\beta_{i,HML}$ & $\beta_{i,RMW}$ are coefficient of risk sensitivity for investment and profitability respectively. The general method of Moments (GMM) is used to estimate the alpha for all three models. GMM applies for estimation as it bypasses the distribution for heteroskedastic and autocorrelation problems for non-normal data also used by (Cochrane, 2009). To check the joint significance of the estimated Coefficient diagnostic test Wald has been applied to see the joint significance for the coefficient. This test applied for both EW and VW returns.

4 | RESULTS and ANALYSIS

4.1 | Preliminary Findings

Table 1

Panel A: Full Sample (JAN 2003-Dec 2021)

Performance of deciles portfolio based on Downside risk												
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	t-stat
Avg D-Beta	0.09	0.16	0.45	0.67	0.89	1.13	1.40	1.73	2.08	2.71	2.62	60.90
EW (% p.a)	-30.27	-2.10	-0.99	-5.35	-1.59	-1.24	2.72	-3.71	-8.21	-5.37	24.90	1.85
VW (% p.a)	-48.07	1.76	4.84	-2.30	-2.37	-4.91	-0.86	-4.54	-4.13	-3.11	44.95	2.27
MV (RS Million)	895.51	3742.	1028	1680	1276	1473	1454	1541	1033	2450	1554.6	7.44
CAPM Beta	-0.18	0.23	0.49	0.56	0.81	1.09	0.95	1.09	1.18	1.32	1.50	5.04

Table 1 shows showing decile portfolio constructed based on downside beta for the time period from (jan2003 dec2021). Ten portfolios are arranged based on lower to higher values of beta. P1 is the portfolio that has the lowest beta value while P10 is the portfolio that has the highest beta value. The P10-P1 column represents the total spread from lowest to highest. It is calculated by subtracting P1 from P10. The last column reports t statistics having a null hypothesis that no difference between the mean of portfolio P1-P10.

Table 2*Panel A: Alphas of Equally Weighted Downside Bet Portfolio (full Sample)*

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	Chi-Sq
CAPM alpha (% p.a)	-0.32	-0.03	-0.03	-0.09	-0.08	-0.10	-0.07	-0.14	-0.20	-0.18	0.14	100.14
	(-6.74)***	(-0.67)	(-0.55)	(-1.79)*	(-1.62)*	(-2.10)**	(-1.36)	(-2.99)***	(-4.23)***	(-3.87)	(2.87)***	(0.00)
Fama French alpha 3	-0.35	-0.01	0.01	-0.05	-0.01	-0.01	-0.03	-0.04	-0.08	0.02	0.37	115.2
	(-10.32)***	(-0.15)	(-0.14)	(-1.32)	(-0.18)	(-0.35)	(-0.85)	(-1.04)	(-2.20)**	(-0.46)	(10.79)***	(0.00)
Fama French alpha 5	-0.37	-0.02	-0.01	-0.08	0.01	-0.05	-0.07	-0.06	-0.05	0.04	0.41	198.2
	(13.10)***	(0.79)	(0.18)	(2.68)**	0.52	(1.79)*	(2.32)**	(2.19)**	(1.67)*	(1.50)	(14.61)***	(0.00)

Table 2 is distributed in two panels; panel A & pane B. Panel A shows alphas for equally weighted downside beta portfolio while Panel B shows the same for Value value-weighted downside portfolio. Table 2 represents the alpha from three different models CAPM, Fama French three & five factor models. Alpha for every model for every portfolio is showing while t-values are showing in brackets () under every coefficient. While the last column represents the joint significance for the entire coefficient. Chi-square statistics refer to the Wald test having a null hypothesis that all alphas are equal to zero. *indicating significance of 10 %; ** indicating significance 5%, *** indicating significance on 1.

Table 3*Panel A: Alphas of Equally Weighted Downside Bet Portfolio (full Sample)*

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	Chi-Sq
CAPM alpha (% p.a)	-0.495	-0.006	0.017	-0.064	-0.081	-0.170	-0.102	-0.161	-0.174	-0.182	0.31	60.8
	(-2.94)***	(-0.10)	0.32	(-0.89)	(-1.57)	(-3.42)***	(-1.85*)	(-3.41)***	(-3.93)***	(-2.56)**	(1.68)*	(0.00)
Fama French alpha 3	-0.50	-0.05	0.00	-0.12	-0.10	-0.25	-0.11	-0.07	-0.08	-0.06	0.44	121.2
	(-5.21)***	(-1.06)	(0.04)	(-2.24)**	(-2.99)**	(-7.14)***	(-3.53)**	(-2.49)**	(-2.85)**	(-1.21)	(3.70)***	(0.00)
Fama French alpha 5	-0.45	-0.02	-0.04	-0.11	-0.13	-0.28	-0.15	-0.11	-0.17	-0.11	0.37	299.5
	(-5.19)***	(-0.71)	(-1.64)	(-2.92)**	(-4.72)**	(-10.74)**	(-5.93)**	(-4.72)***	(-7.59)***	(-2.84)**	(3.65)***	(0.00)

Table 1 shows different descriptive statistics. It contains three panels A, B, and C as the data is distributed in three samples. The first panel shows the statistics of deciles downside beta portfolio for the full sample while in the other two panels' data is divided into two equal periods to compare time factors. There is a significant deviation in downside beta in the portfolio from P1-P10. There is approximately 24 percent of the spread is found in the EW returns while 44 percent spread is estimated in VW returns. This spread refers to portfolio P10 which has the largest value of bate generating more returns as compared to P1 portfolio. When the market is underperforming as compared the investors seek more returns on the security which has a higher downside risk. Results for EW and VW are statistically significant as they show the t values of 24 & 44 respectively. In line with the study of [Harvey and Siddique \(2000\)](#) for US stock and the study of [Chen \(2014\)](#), the various countries also reflect that investor has to require extra incentives to hold the security with higher downside risk. Our findings also suggest that investors are ready to hold the security which has lower downside beta with fewer returns. While there is a side effect found in portfolios. Moreover, once the data sample is distributed in two equal periods then the spread of EW and VW returns becomes insignificant as it shows t-values of 0.39 and 0.37 respectively for the period from 2003-2009 and 2009- 2015. Therefore, we can argue that the returns portfolio after 2009 meets the theories of risk and returns in the portfolios.

However, the findings also suggest that after the financial crisis, the risk-return dynamics shifted, as indicated by the lower t-values of 0.39 and 0.37 in the periods following 2009, implying that the market conditions and investor behavior may have evolved. This shift in risk-return expectations supports the notion

that risk management theories remain pertinent, particularly in changing market environments ([Fisher & D'Alessandro, 2021](#); [Menon et al., 2023](#)).

4.1 | Risk Adjusted Performance

In this section, we estimate the risk-adjusted alphas from CAPM, Fama French 3, and five-factor models. To check the performance of the downside beta's portfolio. The estimates are being calculated for equally weighted and value-weighted portfolios. Full and partial samples are used to see the time effect as well. Panel A of Table 1 reports the significant spread of portfolio 10 to portfolio 1 as it is 24 percent for equally weighted and 44 for Value weighted portfolio which clearly shows that investors hold a portfolio of higher risk with the highest return. They are also ready to hold a portfolio of lower risk with low returns which aligns with the theory of risk and return. While the market value of portfolios does not impact the largely as there is no certain pattern in capture that can play the role. Moreover, CAPM betas for all portfolios are also significant in explaining the risk-return pattern between the ten portfolios. While in the partial sample period presented in Panels B and C does not show in difference in the significance level, although there is a bit increase in the market value of Portfolio 10 after 2009. This may happen due to the effect of the subprime mortgages on the Pakistan stock market. Where people might be affected by the risk factor and start investing in big firms.

As we calculated the alpha value using CAPM, the Fama French model is reported in Tables 2 & 3 respectively. CAPM and Fama French 3 and 5-factor alphas for equally weighted portfolios are significant in both full and partial sample periods. However, Value value-weighted portfolio does not report significance in the partial sample period. More specifically, the Fama French alpha of the 3-factor model is insignificant in the partial period as it shows the t-statics .037 and .040. In more precise all three models rejected the hypothesis that alpha is equal to zero. Therefore, we can argue based on our finding that the Mean-variance framework could not possibly well explain the downside risk so we may go for another model to explain the better relationship of downside risk.

5 | CONCLUSION

After careful consideration of the estimates of all mentioned models, there should be significant effects of downside beta in the portfolio. Portfolio spread showing significant results. Portfolio 10 has the highest beta value reporting the highest return value which alienates the theoretical aspects of the risk and return relationship. CAPM betas also reported a significant result in the portfolio. To compare the beta and downside beta values in terms of magnitude then the downside beta values are a bit higher than normal beta. Furthermore, the risk-adjusted return shall be estimated through CAPM and Fama French 3 and 5-factor model. Results show that the alpha value is significant for the entire model which rejects the hypothesis that alpha is equal to zero. Although the joint significant of coefficient i tested through the Wald test which reported the significant results and rejection of the hypothesis that all portfolios mean are equal. Results indicating the entire portfolio has a different mean thus their construction is effective. Eventually, the findings of this research will give us the path to apply some other models like higher moments which may explain downside risk better than the mean variance approach.

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